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A MICROSCOPIC TEST OF INVERTED NUCLEAR COEXISTENCE

by Richard C. Braley and William F. Ford
Lewis Research Center
Cleveland, Ohio

TECHNICAL PAPER presented at
American Physical Society Meeting
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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ABSTRACT

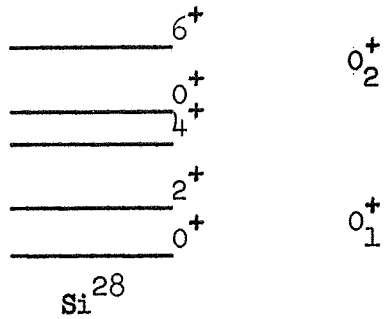
It is well known that the Hartree-Fock method of deformed orbitals is less successful for nuclei in the upper half of the 2s-1d shell than for those in the lower half. Of the nuclei in the upper half, Si^{28} and S^{32} have received perhaps the greatest attention. Certain features of their spectra motivated Bar-Touv and Goswami to investigate the possibility of inverted coexistence in these nuclei.[1] The results of their study indicate that a more detailed investigation of this question is warranted. In our study, the ground state intrinsic wavefunctions for Si^{28} and S^{32} are each assumed to be a spherical plus deformed Slater determinant. States of good angular momentum are obtained by projection. Several nuclear properties such as radii, transition rates, electron scattering, form factors, and nucleon scattering cross sections are calculated and compared with experiment.

INTRODUCTION

The Hartree-Fock (HF) method has contributed greatly to our understanding of the underlying microscopic structure of deformed nuclei. Structure calculations which make use of the HF method have been most promising in the lower half of the 2s-1d shell.( ) In this region, the N = Z open shell nuclei Ne^{20} and Mg^{24} have received the greatest attention. Projected HF wave functions have been used quite successfully to study many properties of these nuclei, as well as others.[2] Unfortunately the structure studies, as well as inelastic scattering studies in upper half of the 2s-1d shell have not met with the same degree of success. In particular, the nuclei Si^{28} and S^{32} have been studied extensively and it is clear at this point that the simple (single-Slater deteminant) HF picture is inadequate for these nuclei.

Recently Bar-Touv and Goswami investigated the possibility of "inverted coexistence" of spherical and deformed states in these nuclei (as well as C^{12}).[1] Their predictions are based on the observation that closed subshell nuclei (such as Si^{28} and S^{32}) possess spherical HF solutions in addition to the lower deformed solutions which are usually found. The existence of such spherical states in N = Z closed subshell nuclei manifests itself in the appearance of an excited 0^+

state embedded in the ground state rotational band. (This is referred to as inverted coexistence.) In Fig. 1 is shown part of the spectrum of Si^{28}, demonstrating the type of energy level systematics suggested by Bar-Touv and Goswami for these nuclei.



Their results are in excellent agreement with the experimental spectra and ratios of E2 transition rates. Thus, it is indicated that the idea of inverted coexistence for these nuclei is a valid one, and that a more detailed investigation of the problem is warranted.

It is the purpose of this paper to provide such an investigation. The ground state intrinsic wave functions for the nuclei of interest are assumed to be linear combinations of spherical and deformed Slater determinants. States of good angular momentum are obtained by the standard projection technique. Radii, E2 rates, elastic electron form factors and nucleon scattering cross sections are calculated and compared with experiment.

Theory

The spherical and axially-symmetric (deformed) intrinsic states are obtained using the HF method. Physical states of interest are obtained by projection.[3] All of the calculations are performed with the Tabakin separable potential,[ ] using basis functions with harmonic oscillator radial dependence and span the 1s, 1p, 2s-1d, 2p-1f, and 3s-2d-1g shells.

The observables which will be studied (B(E2), nuclear radius, $d\sigma/d\Omega$) are related to the reduced matrix elements of single-particle operators, i.e., operators of the form

$$\Omega = \sum_{n=1}^{A} \Omega(\vec{x}_n) \qquad (1)$$

Hence the nuclear matrix elements can be expressed in terms of single particle matrix elements:

$$\langle f\|\Omega\|i\rangle_{JM} = \sum_{a,b} S_J(if|ab)\ \langle b\|\Omega\|a\rangle_{JM} \tag{2}$$

Here the labels a and b refer to the basis functions. The quantities $S_J(if|ab)$ have been discussed in detail elsewhere for the case in which the states i and f are obtained from single Slater determinants.[3] For the case of interest here, the initial state (ground state) is a sum of spherical and deformed states. The spherical state is

$$\psi_s = (A!)^{-1/2}\ \det\{\varphi_\mu\} \tag{3}$$

where the orbits Q_μ are spherical. The deformed state is obtained using standard projection techniques:.

$$\psi_D = P^J_{MK}\Phi_K \tag{4}$$

Φ is the intrinsic deformed HF determinant. So the initial state is

$$\psi_{g.s} = \alpha\psi_S + \beta\psi_D \tag{5}$$

When the final nuclear state is obtained from a single Slater determinant, it can be shown that

$$S_J(if\ ab) = \frac{\alpha\ S_J(i_S f|ab) + \beta\ S_J(i_D f|ab)}{\left[\alpha^2 + \beta^2 + 2\alpha\beta \sum_c S_0(i_S i_D|cc)\right]^{1/2}} \tag{6}$$

where i_S and i_D refer to the spherical and deformed components, respectively, of the initial state. The denominator in Eq. (6) results from the normalization of $\psi_{g.s.}$. If the final state is also

a linear combination of two states, then Eq. (6) may be applied to the components of the final state, to construct the S_J's. Once the S_J's are obtained, reduced matrix elements may be calculated using Eq. (2).

RESULTS AND DISCUSSION

The deformed HF solutions for both of the nuclei considered here correspond to oblate intrinsic shapes. However in the case of ^{32}S the prolate solution is almost degenerate with the oblate, hence there is no a priori reason for disregarding such a solution.

In table I we compare the results for the predicted E2 rates and RMS radii with experiment. For both nuclei, the calculated RMS, radii are excellent agreement with experiment. The purely deformed solutions over estimate the E2 rates for $(0_1^+ - 2_1^+)$, which is very unusual. One generally expects that E2 rates will be underestimated. When the ground state wave function is assumed to be of the inverted coexistence type suggested by Bar-Touv and Goswami (I.C.), it is seen that the $B(E2; 0_1^+ - 2^+)$ rates are brought into substantially better agreement with experiment. However, examination of the $B(E2; 0_2^+ - 2^+)$ are seen to be badly underestimated. This, of course, indicates that the 0_2^+ states are considerably more collective than one could obtain from a purely spherical state. This prompted us to consider the 0_2^+ states in ^{28}Si and ^{32}S to also be a linear combination similar to (5); i.e.,

$$\psi_{exc} = \alpha'\psi_S + \beta'\psi_D \qquad (7)$$

where α' and β' are determined by the condition that

$$\langle\psi_{exc}|\psi_{g.s.}\rangle = 0, \qquad \text{and} \qquad \langle\psi_{exc}|\psi_{exc}\rangle = 1.$$

The results obtained using (5) and (7) appear in the row (I.C.)* of table I. while the $B(E2; 0_1^+ - 2^+)$ are almost unaffected, the transition rates from the 0_2^+ to the 2^+ state are improved quite significantly.

The inelastic scattering of nucleons provides an additional test of the nuclear wave function - particularly in the region of the nuclear surface. The results of DWBA predictions for the $(0_1^+ - 2^+)$ inelastic

proton cross sections appear in Figs. 1 and 2.[4] The ground state and excited state for each nucleus are represented by Eqs. (5) and (4) respectively. The optical model parameters used in the DWBA problem yield very good fits to the elastic scattering; the theoretical results have been multiplied by two in order to demonstrate that for most of the angular range the result is quite good. However, for angles less than 40° the distributions break from the experimental shapes. The fact that the 0_2^+ and other low-lying levels are so strongly excited indicate that a coupled channel study should be made for these nuclei.

In Figs. (3) and (4) we present the elastic electron scattering form factors. The theoretical results are based on the solution of the Dirac equation in the Glauber approximation and was originally developed by Baker.[5] The nuclear ground state is represented by (5). Based on these results, it would seem that the nuclear interior is not well represented by our wave functions. On the other hand it may be that the difficulty lies with the treatment of the scattering problem. This question can only be answered by an exact phase shift analysis.

Further study of the structure of these nuclei will require projected energy spectra. This is an extensive problem for such a large basis (15 states), and is presently being undertaken.

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TABLE I

Nucleus	Model†	$\langle R^2 \rangle^{1/2}$ fm	$B(E2; 0_1^+ - 2_1^+)$, $e^2 \cdot fm^4$	$B(E2; 0_2^+ - 2_1^+)$, $e^2 \cdot fm^4$
Si^{28}	D.	3.096	376.2	
	I.C.	3.089	339.5	1.63
	(I.C.)*	3.092	342.3	34.2
	Exp't	3.08 ± 0.06	327 ± 17	35 ± 4
S^{32}	D.	3.246	244.2	
	I.C.	3.238	179.9	22.0
	Exp't	3.23 ± 0.07	217 ± 30	110 ± 9
	(I.C.)*	3.248	181.2	72.2

†D. corresponds to the purely deformed solution,
I.C. refers to inverted coexistence as suggested by Bar-Touv and Goswami.( )
(I.C.)* refers to the model suggested by the present authors.

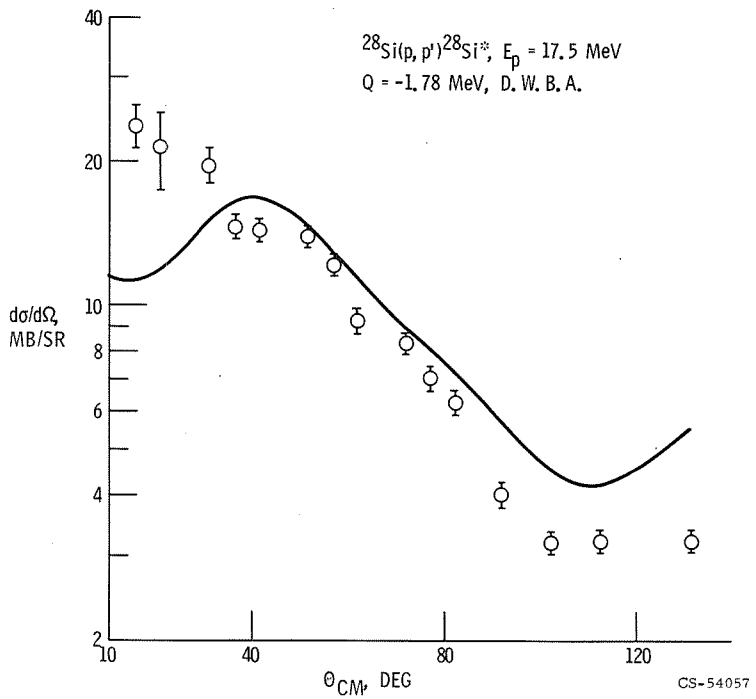

Figure 1

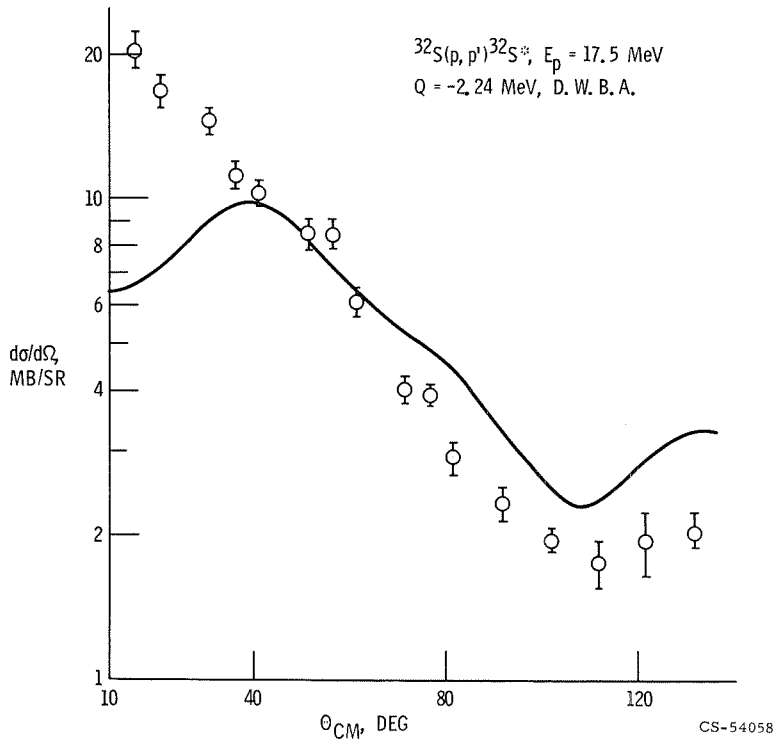

Figure 2

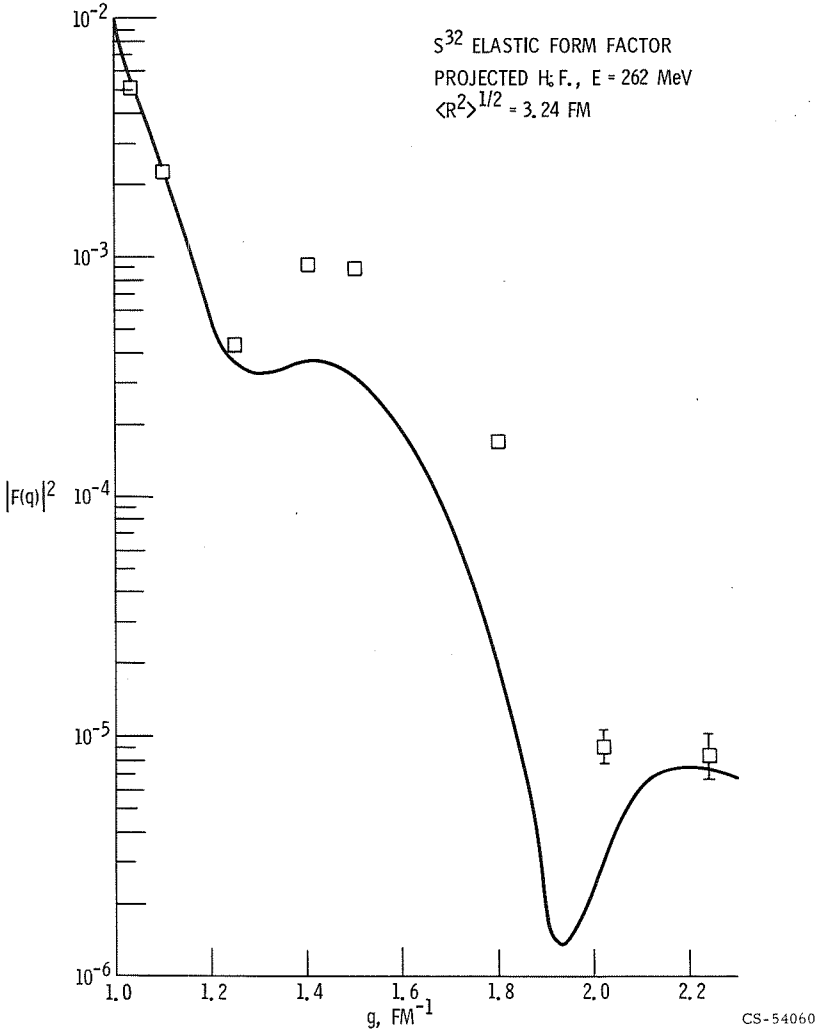

Figure 4

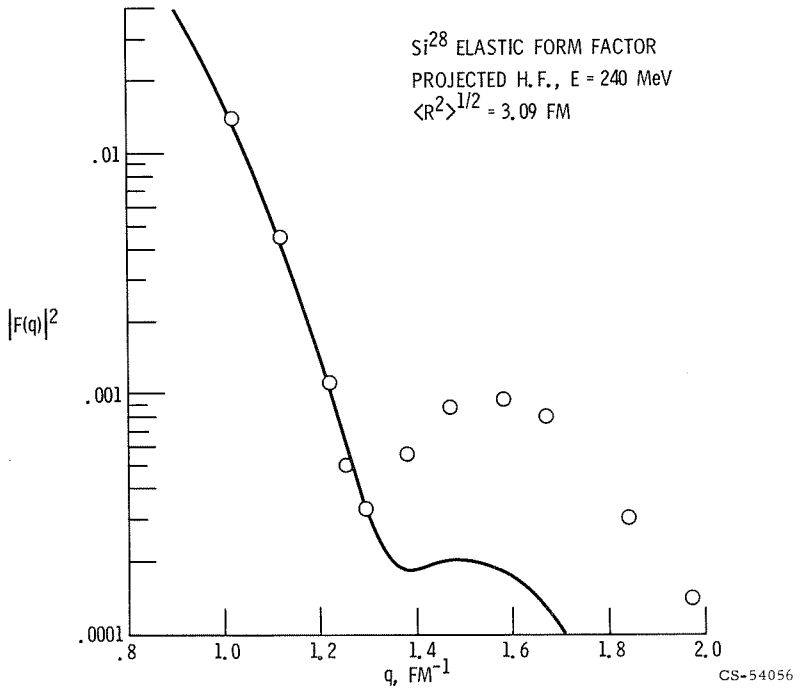

Figure 3

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